JOINT INVENTORS

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APPLICATION FOR
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SPECIFICATION

TO ALL WHOM IT MAY CONCERN:

Be it known that we, **Achim Schweikard**, a citizen of Germany, residing at Wrangel Str 28, Hamburg 20253 and **Stefan Burkhardt**, a citizen of Germany, residing at Welschstrasse 7, Munchen 81929 have invented a new and useful **Correcting Geometry and Intensity Distortions in MR Data**, of which the following is a specification.

Correcting Geometry and Intensity Distortions in MR Data

5

BACKGROUND OF THE INVENTION

Technical Field

10 The invention relates to the field of Magnetic Resonance Imaging (MRI). In particular, the invention relates to a method, a computer program product and an apparatus which allow the correction of geometrical and intensity distortions inherent in Magnetic Resonance (MR) data.

15 ### Description of the Prior Art

MRI is a powerful technology for acquiring images with high tissue contrast. Besides the high tissue contrast, the potential for tumor localization and the possibility to scan in any plane orientation have made MRI a useful tool in many fields of medi-
20 cine.

MRI relies on the principle that an arbitrary object of interest is magnetized by a strong and homogenous static magnetic field B_0. The homogeneity of the static magnetic field B_0 is a very important aspect of MRI because any perturbations of the
25 homogeneity lead to geometry and intensity distortions in the image plane as well as to displacement, warp and tilt of the image plane itself.

In reality, the static magnetic field B_0 is never homogeneous but perturbed. One reason for perturbations of the magnetic field B_0 is the object of interest itself which
30 is placed in the magnetic field B_0.

When an object having a specific magnetic susceptibility distribution $\chi(\mathbf{x})$ is placed in the homogeneous and static magnetic field B_0, the object becomes magnetized and the homogenous static magnetic field B_0 is distorted giving rise to an induced
35 magnetic field **B**. For an analysis of the geometry and intensity distortions caused by an object placed in the homogeneous static magnetic field B_0, the field **B** has to be determined.

In order to determine **B**, the Maxwell equations have to be solved. For a magneto-static problem the Maxwell equations reduce to the Laplace equation

$$div(\mu\Delta\Phi_M) = 0 .$$

<div align="right">(1)</div>

Here Φ_M is the magnetic scalar potential in [Wb/m] and $\mu = 1 + \chi$ is the dimensionless magnetic permeability. If the susceptibility distribution $\chi(\mathbf{x})$ of an object is known, Φ_M is determined by solving equation (1). From Φ_M the magnetic field **H** in [H/m]

$$\mathbf{H} = -\nabla\Phi_M$$

<div align="right">(2)</div>

and the induced magnetic field **B** in [T]

$$\mathbf{B} = \mu_0\mu\,\mathbf{H}$$

<div align="right">(3)</div>

can be deduced. μ_0 denotes the permeability of vacuum and has a value of $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Equation (1) can be solved analytically for very simple objects such as cylinders and spheres. For more complex objects equation (1) can be solved only numerically. An exemplary numerical analysis of the magnetic field **B** for arbitrary magnetic susceptibility distributions $\chi(\mathbf{x})$ in two and three dimensions is discussed in R. Bhagwandien: "Object Induced Geometry and Intensity Distortions in Magnetic Resonance Imaging", PhD thesis, Universiteit Utrecht, Faculteit Geneeskunde, 1994, ISBN: 90-393-0783-0.

Susceptibility related distortions in MRI are usually in the millimeter range and have therefore no influence on diagnostic applications. However, in certain applications like Radio Therapy Planning (RTP) the geometric accuracy of an MR image is of high importance because accurate beam positioning is essential for optimal tumor coverage and sparing healthy tissues surrounding the tumor as much as possible.

Based on a numerical solution of equation (1), various methods have been proposed to reduce susceptibility induced distortions in MR images.

In the Bhagwandien document a correction method is described which is based on
just one image, namely the distorted MR image. According to this correction method,
the distorted MR image is first converted into a magnetic susceptibility distribution by
segmenting the MR image into air and water equivalent tissue. In a next step the
susceptibility distribution thus obtained is used to numerically calculate the field **B**.
Finally, the corrected MR image is calculated on the basis of a read out gradient that
is reversed with respect to the read out gradient used to acquire the distorted MR
image. If for example a gradient field of a specific strength G_z has been applied
during acquisition of the distorted MR image to define the image plane in z-direction,
the corrected MR image is calculated for a gradient field of the strength $-G_z$.

A major draw back of all methods hitherto used to correct distortions in MR data is
the computational complexity involved in generating corrected MR data. Conse-
quently, there is a need for a method, a computer program product and an apparatus
for correcting distortions in MR data faster.

SUMMARY OF THE INVENTON

This need is satisfied according to the invention by a method of correcting distortions
in MR data, the method comprising the steps of providing distorted MR data of an
object of interest and distortion parameters for one or more generic objects,
determining transformation parameters correlating the object of interest and one or
more of the generic objects, and processing the distorted MR data taking into
account the distortion parameters and the transformation parameters to obtain
corrected MR data.

By using generic distortion parameters, i.e. distortion parameters derived for generic
objects, the corrected MR data for the object of interest can be generated faster. The
reason for this is the fact that the Laplace equation (1) has not necessarily to be
solved individually for every set of distorted MR data.

The distortion parameters for a particular generic object may be determined in
various ways. The distortion parameters for a particular generic object may for
example be derived from magnetic field inhomogenities which result from the specific
magnetic susceptibility distribution of this particular generic object when the object is
placed in a homogeneous static magnetic field. According to a first variant, the
magnetic field inhomogenities caused by the generic object are determined by way

of measurements. According to a second variant, the magnetic field inhomogenities are derived by way of calculations from distorted MR data of the generic object, i.e. from distorted generic MR data.

5   Preferably, the magnetic field inhomogenities, i.e. the distortion parameters, are derived from distorted generic MR data. Generic MR data may be obtained for e.g. a specific part of the human body from commercial databases. However, the generic MR data may also be generated using available generic objects during a data acquisition phase preceding the actual acquisition of the distorted MR data of the object of
10   interest.

Deriving the magnetic field inhomogenities from generic MR data may include two separate steps. In a first step the magnetic susceptibility distribution of the generic object may be determined from the distorted generic MR data. To that end an image
15   generated on the basis of the distorted generic MR data may be segmented automatically or manually to obtain areas of common or similar magnetic susceptibility. Then, an appropriate susceptibility value may be automatically or manually assigned to each area having the same or a similar magnetic susceptibility.

20   Once the magnetic susceptibility distribution of the generic object has been determined, the magnetic field inhomogenities are derived from the determined susceptibility distribution in a second step. The second step may include a numerical approach in order to solve the Laplace equation (1) for the determined susceptibility distribution. The numerical approach may for example be based on transforming the
25   Laplace equation (1) into a diffusion equation and on solving this diffusion equation by means of a diffusion technique. Preferably, however, a multi-grid approach is used to solve the Laplace equation (1) for the determined susceptibility distribution. By means of a multi-grid algorithm the computational complexity is reduced since the iterations that normally take place on a fine grid are replaced by iterations on a
30   coarser grid.

The multi-grid approach is not restricted to solving the Laplace equation (1) in context with determining the distortion parameters for a generic object but can directly be applied to correct distortions in MR data "on-line". According to this "on-
35   line" aspect of the invention, the distorted MR data of the object of interest are first converted into a magnetic susceptibility distribution and the susceptibility distribution thus obtained is used to numerically calculate (using the multi-grid approach) the magnetic field B induced by the object of interest. The corrected MR data of this

object may then be calculated on the basis of a read out gradient (i.e. gradient field strength) that is reversed with respect to the read out gradient used to acquire the distorted MR data of the object of interest.

5   In the course of correcting the distorted MR data of the object of interest a correlation between the object of interest and one or more of the generic objects has to be established. To this end transformation parameters indicative of the correlation are determined. The transformation parameters indicate how the object of interest will be deformed during a mapping operation on the one or more generic objects or how

10   the one or more generic objects will be deformed during a mapping operation on the object of interest. Preferably, the transformation parameters are determined for specific points, contours, areas or other features which the object of interest and the one or more generic objects have in common.

15   The transformation parameters may be determined on the basis of magnetic susceptibility data of the object of interest and of the one or more generic objects. Preferably, the magnetic susceptibility data are derived from distorted MR data by e.g. segmenting the distorted MR data into regions of changing magnetic susceptibility (magnetic susceptibility contours) or regions (areas) of common or similar magnetic

20   susceptibility. After the distorted MR data of the object of interest have been segmented, transformation parameters are derived which deform at least one segmented region (e.g. a specific area or a specific contour) determined from the distorted MR data of the object of interest onto a corresponding region of one or more of the generic object or vice versa.

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Once appropriate transformation parameters have been determined, the distorted MR data of the object of interest are corrected. To that end, the distortion parameters for the one or more generic objects and the transformation parameters may be used for calculating distortion parameters for the object of interest. Since the calcula-

30   tion of the distortion parameters for the object of interest involves a mere correlation of the distortion parameters for the one or more generic objects and the transformation parameters, no differential equation has to be solved. The distortion parameters for the object of interest can thus be obtained in a fast and easy manner. Once the distortion parameters for the object of interest are known, they may be reverse-

35   applied to the distorted MR data of the object of interest or to data derived therefrom to obtain corrected MR data.

For a particular generic object several sets of distortion parameters may be provided for different gradient field strengths, different phase encoded directions, etc. This enables to select the set of distortion parameters that corresponds with respect to gradient field strength, phase encoded direction, etc. to the gradient field strength, phase encoded direction, etc. used while generating the distorted MR data of the object of interest.

MR data of a phantom object may be co-generated with the distorted MR data of the object of interest. By using a phantom object like a sphere or a cylinder with known characteristics it is possible to estimate the gradient field strength that was used while generating the distorted MR data of the object of interest. As has been mentioned above, knowledge of the gradient field strength used while generating the distorted MR data of the object of interest is of importance e.g. if generic distortion parameters for different gradient field strengths are available.

Moreover, MR data of a phantom object co-generated with the distorted MR data of the object of interest allow to redundantly verify the correction of the distorted MR data. If for example the relationship between distorted and corrected MR data of the phantom object is known, this relationship may be compared to the relationship between the distorted and the corrected MR data of the object of interest. By comparing the two relationships a quality parameter for the corrected MR data of the object of interest can be derived.

Since the invention allows to correct geometry and intensity distortions in a fast and reliable manner, it is advantageous to repeatedly acquire and correct distorted MR data of the object of interest at different points in time. Due to the high reliability of the corrected MR data, chronological changes of the object of interest can be assessed. This enables for example to detect tumor growth in the sub-millimeter range. For this purpose MR data generated for one and the same object of interest at different points in time may be registered relative to each other. In the case of MR image data, the registration might result in a spatial superposition of the individual sets of MR image data.

According to a preferred aspect of the invention the corrected MR data of the object of interest are combined with Computer Tomography (CT) or fluoroscopic data of the object of interest to profit from the individual advantages of each imaging technology. For example the CT or fluoroscopic data and the corrected MR data of the object of interest may be registered relative to each other and the registered data may be

used for generating an image showing a combination of the structures of the object of interest that were detected by MR on the one hand and by e.g. CT on the other hand.

5   Preferably, the corrected MR data or data derived therefrom like CT or fluoroscopic data registered with the corrected MR data are used for generating a graphical display for example on a display device of a computer system. The corrected MR data or the data derived therefrom may then be used for navigating a chirurgical tool or a pointer during e.g. bone surgery. To that end, an infrastructure may be provided
10   which allows to determine on-line the current position of at least one of the surgical tool and the pointer with respect to the corrected MR data or the data derived therefrom. By superimposing the current position of at least one of the surgical tool and the pointer on the graphical display of the corrected MR data or the data derived therefrom a navigation aid for the surgical tool or the pointer is achieved. In the
15   context of navigating a surgical tool or a pointer the fact that geometry and intensity distortions in the MR data of the object of interest have been corrected is of high importance because otherwise an exact navigation would not be possible.

The method according to the invention can be implemented as a hardware solution
20   and as a computer program product comprising program code portions for performing the individual steps of the method when the computer program product is run on a computer. The computer program product may be stored on a computer readable recording medium like a data carrier attached to or removable from the computer.

25   The hardware solution includes an apparatus comprising a common database or separate databases for at least temporarily storing distorted MR data of the object of interest and distortion parameters determined for one or more generic objects, a transformation parameter generator for generating transformation parameters correlating the object of interest and one or more of the generic objects, and a
30   generator for corrected MR data for processing the distorted MR data taking into account the distortion parameters and the transformation parameters to obtain corrected MR data for the object of interest. Preferably, the apparatus is part of a navigational infrastructure for computer aided surgery.

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BRIEF DESCRIPTION OF THE DRAWINGS

Further aspects of the invention will become apparent from the following description of various embodiments of the invention and upon reference to the drawings, in which:

Fig.1   is a schematic diagram of an apparatus according to the invention for correcting distortions in MR data;

Fig. 2   is a block diagram depicting the steps performed according to the invention during the correction of distorted MR data;

Fig. 3   schematically shows a correlation between fine and coarse grids performed by a multi-grid algorithm according to the invention;

Fig. 4   schematically shows the procedure used to calculate Φ_M;

Fig. 5A and 5B   show the results of MR/CT registration using distorted MR data on the one hand and corrected MR data on the other hand;

Fig. 6   shows a curve of mutual information during the MR/CT registration in a region around the global maximum;

Fig. 7   is a block diagram depicting the steps performed according to the invention when applying a multi-grid algorithm to a susceptibility distribution derived for the distorted MR data of the object of interest; and

Fig. 8   shows a navigational infrastructure according to the invention.

DESCRIPTION OF PREFERRED EMBODIMENTS

In the following, the invention will exemplarily be set forth with respect to correcting distorted MR image data in context with tumor diagnosis and bone surgery. It should be noted, however, that the invention could be implemented in any technical field like material sciences requiring the generation of highly accurate MR data.

Hardware Aspects

In Fig. 1 an apparatus 10 according to the invention for correcting distortions in MR data is depicted. The apparatus 10 may for example be configured as a computer terminal and comprises a database 12 for storing newly acquired distorted MR data of an object of interest like a patient's head. Additionally, a plurality of sets of generic distortion parameters which have previously been generated as will be discussed below with reference to Fig. 2 are stored in the database 12. Furthermore, for each set of generic distortion parameters and/or each generic object an individual generic susceptibility distribution is stored in the database 12.

A transformation parameter generator 14 of the apparatus 10 is configured such that it has access to the database 12 and in particular to the distorted MR data of the object of interest and the generic susceptibility distributions stored therein. The transformation parameter generator 14 is capable of determining the susceptibility distribution of the object of interest and of selecting the susceptibility distribution of this generic object which matches the object of interest best. By correlating the susceptibility distribution of the object of interest and the selected generic susceptibility distribution, the transformation parameter generator 14 determines a set of transformation parameters which are output to a generator 16 for corrected MR data.

The generator 16 for corrected MR data is configured to access the database 12 and to read out the distorted MR data of the object of interest and the generic distortion parameters corresponding to the particular generic susceptibility distribution selected by the transformation parameter generator 14. Taking into account the transformation parameters determined by the transformation parameter generator 14 and the specific generic distortion parameters obtained from the database 12, the generator 16 for corrected MR data processes the distorted MR data of the object of interest to generate corrected MR data of the object of interest as will be explained below in more detail.

The corrected MR data of the object of interest determined by the generator 16 are output to a display device 18 which transforms the received MR data into corresponding image data and generates a graphical display of the corrected MR data of the object of interest.

In the following, the individual steps performed according to the invention during the correction of distorted MR data of an object of interest are explained in more detail with reference to Figs. 2 to 7.

5   <u>Generation of Input Data</u>

Referring to Fig. 2, a first set of input data is constituted by newly acquired distorted MR data of an object of interest, e.g. a patient's head. The distorted MR data of the object of interest are acquired using an MR scanner as is known in the art. Besides

10   the distorted MR data of the object of interest, the gradient field strength and the phase encoded direction employed during the data acquisition process for the object of interest are stored at least temporarily also.

In addition to the distorted MR data of the object of interest, a plurality of distorted

15   generic MR data for several different generic objects are provided. For each generic object, several sets of generic MR data acquired at different gradient field strengths and phase encoded directions may be available.

The individual sets of generic MR data can for example be obtained from a commer-

20   cially available anatomical atlas of from specific databases. In principle, the sets of distorted generic MR data may also have been generated by the MR scanner which is used to acquire the distorted MR data of the object of interest.

Each set of generic MR data comprises a DICOM header which contains information

25   about the gradient field strength and the phase encoded direction employed during the acquisition of the specific generic MR data.

<u>Determination of the Susceptibility Distributions</u>

30   The distorted MR data of the object of interest and the individual sets of distorted generic MF data are processed to determine the magnetic susceptibility distribution of the object of interest and the generic objects, respectively.

Usually, processing of the sets of generic MR data will have taken place prior to the

35   acquisition of the distorted MR data of the object of interest. In other words, there may be several sets of generic susceptibility distribution data already readily available (e.g. stored in the database 12 of the apparatus 10 depicted in Fig. 1) when the correction of the distorted MR data of the object of interest is started.

The process of determining the magnetic susceptibility distribution is the same for the distorted MR data of the object of interest and the distorted generic MR data. During the step of determining the magnetic susceptibility distribution, an MR image

5 corresponding to the MR data is generated and manually or automatically segmented into areas corresponding to air and areas corresponding to different tissues like bone, muscle and fat. This might involve a determination of the outer contour of the individual object.

10 After the segmentation step, each individual segment is assigned an appropriate magnetic susceptibility. The magnetic susceptibilities is of materials like human tissue are well known.

Determination of Transformation Parameters

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The determination of the transformation parameters starts with selecting the generic susceptibility distribution of this generic object which matches the object of interest best. If the object of interest is e.g. a patient's head, the generic susceptibility distribution of a generic patient's head having the same size or diameter can be

20 selected. Additionally, a patient's sex, age, etc. can be taken into account. Thus, the generic object which matches best the object of interest has to be determined.

Since for each generic object distorted MR data acquired at different gradient field strengths and phase encoded directions may be available, this particular generic

25 susceptibility distribution is selected which corresponds with respect to gradient field strength and phase encoded direction to the distorted MR data of the object of interest.

Once an appropriate generic susceptibility distribution has been selected, the genera-

30 tion of the transformation parameters continues with marking or segmenting prominent points, contours or areas in the susceptibility distribution of the object of interest. The selected generic susceptibility distribution is subjected to a similar processing to identify the prominent point, contours and areas which are to be associated with the corresponding features of the susceptibility distribution of the

35 object of interest.

On the prominent points, contours or areas of both the susceptibility distribution of the object of interest and a selected generic susceptibility distribution control points

are set. By means of an optimization procedure a deformation for one of the two distributions is then determined which minimizes the distance between the corresponding control points of each distribution. To this end thin-plate splines, bilinear interpolation, deformation procedures based on Gauss' least square optimization or

5   any similar deformation procedures or combinations thereof may be used.

According to an alternative approach, the deformation is performed without any control points. In such a case the respective grey value or color value distribution of images corresponding to the individual susceptibility distributions are used. The

10   matching between two corresponding susceptibility distributions can be assessed after an iteratively performed deformation step by means of procedures of mutual information, cross correlation procedures or similar procedures or combinations thereof. The optimization of a deformation is thus based on a rating function which measures the degree of matching between two images.

15

Once an optimum deformation has been found by any of the procedures mentioned above, the corresponding transformation parameters can be determined by assessing the deformation required to map the susceptibility distribution of the object of interest onto the corresponding generic susceptibility distribution. It should be noted

20   that deviating from the steps mentioned above the transformation parameters may also be derived directly from a distorted MR image of the object of interest and the generic MR image of an appropriate generic object which have been acquired at a similar field strength and phase encoded direction.

25   <u>Determination of the Generic Distortion Parameters</u>

The determination of the generic distortion parameters starts with the calculation of the magnetic field **B** associated with the selected generic susceptibility distribution. To that end the Laplace equation (1) has to be solved. Before equation (1) can

30   numerically be solved it has to be transposed to a finite difference equation. After that it can be solved using iterative approaches (e.g. Jacobi or Gauss-Seidel iterations) or a multi-grid algorithm.

Resolving of the equation (1) with substitution of the div- and ∇-operators leads to

35   the equation

$$\frac{\partial^2 \Phi_M}{\partial x^2} + \frac{\partial^2 \Phi_M}{\partial y^2} + \frac{\partial^2 \Phi_M}{\partial z^2} + \frac{1}{\mu}\frac{\partial \mu}{\partial x}\frac{\partial \Phi_M}{\partial x} + \frac{1}{\mu}\frac{\partial \mu}{\partial y}\frac{\partial \Phi_M}{\partial y} + \frac{1}{\mu}\frac{\partial \mu}{\partial z}\frac{\partial \Phi_M}{\partial z} = 0$$

(4)

Equation (4) is a partial difference equation of elliptic type. The solution of this equation is restricted to the grid points which are given by the MR data, i.e. the MR image. These points are

$$x_i = x_0 + i \cdot \Delta_x,$$

(5)

$$y_i = y_0 + j \cdot \Delta_y,$$

(6)

$$z_i = z_0 + k \cdot \Delta_z.$$

(7)

$\Delta_x, \Delta_y, \Delta_z$ denote the grid width in $x-, y-$ and z-direction. The values of Φ_M will only be computed at these points.

Resolving equation (4) while restricting the solution to the above grid points leads to

$$\frac{\Phi_{i-1,j,k} - 2\Phi_{i,j,k} + \Phi_{i+1,j,k}}{\Delta_x^2} + \frac{\Phi_{i,j-1,k} - 2\Phi_{i,j,k} + \Phi_{i,j+1,k}}{\Delta_y^2} + \frac{\Phi_{i,j,k-1} - 2\Phi_{i,j,k} + \Phi_{i,j,k+1}}{\Delta_z^2} +$$

$$a\frac{\Phi_{i+1,j,k} - \Phi_{i,j,k}}{\Delta_x} + b\frac{\Phi_{i,j+1,k} - \Phi_{i,j,k}}{\Delta_y} + c\frac{\Phi_{i,j,k+1} - \Phi_{i,j,k}}{\Delta_z} = 0$$

(8)

The value $\Phi_{i,j,k}$ of boundary voxels is given. a, b and c represent the values of the partial derivates of μ which are calculated using a difference scheme:

$$a = \frac{1}{\mu}\frac{\mu_{i+1,j,k} - \mu_{i,j,k}}{\Delta_x}$$

$$b = \frac{1}{\mu}\frac{\mu_{i,j+1,k} - \mu_{i,j,k}}{\Delta_y}$$

$$c = \frac{1}{\mu}\frac{\mu_{i,j,k+1} - \mu_{i,j,k}}{\Delta_z}$$

(9)

Equation (8) can be solved iteratively using e.g. a Jacobi or Gauss-Seidel scheme, but usually these methods take a large number of iterations to converge. If only Jacobi- or Gauss-Seidel iterations were used to solve for the equation, the approach would be very slow for large data amounts, i.e. large images.

One may also solve equation (8) using a multi-grid algorithm. Thus, computational complexity may be reduced by replacing the iterations that would normally take place on a fine grid by iterations on a coarser grid. This has the advantage that the time complexity is only linear with respect to the number of observed grid points. In this section it will be briefly reviewed how Gauss-Seidel iterations are performed and used within a multi-grid algorithm.

Equation (8) can be written in terms of a matrix equation

$$\mathbf{A}\,\Phi = \mathbf{f}$$

$$(10)$$

where $\mathbf{A}$ denotes the coefficients, Φ the values of $\Phi_{i,j,k}$ and $\mathbf{f}$ the right side of equation (10) stacked in vector form (in this case $\mathbf{f} = 0$).

$\mathbf{A}$ is decomposed such that $\mathbf{A} = \mathbf{D} - \mathbf{L} - \mathbf{R}$, where $\mathbf{D}$ is diagonal, $\mathbf{L}$ is lower triangular and $\mathbf{R}$ is upper triangular. Now the Gauss-Seidel iteration is given by

$$\Phi^{(i+1)} = (\mathbf{D} - \mathbf{L})^{-1}(\mathbf{f} + \mathbf{R}\Phi^{(i)})$$

$$(11)$$

with any initial value $\Phi^{(0)}$.

The basis of the multi-grid algorithm is the coarse grid correction scheme, where an approximate solution is refined by solving for the error on a coarse grid and updating it with the error.

After i Gauss-Seidel iterations one gets an approximate solution $\Phi^{(i)}$ for the equation. The remaining error is defined as

$$\mathbf{e} = \Phi - \Phi^{(i)}$$

$$(12)$$

and the residual **r** is given by

$$\mathbf{r} = \mathbf{f} - \mathbf{A}\Phi^{(i)}.$$

(13)

Then the error satisfies the equation

$$\mathbf{Ae} = \mathbf{r}$$

(14)

which can be solved by Jacobi or Gauss-Seidel iterations too. However, this equation will be solved approximately on a grid with a coarser resolution. Therefore two operators are defined, a restriction operator $\mathcal{R}$ which transforms the residual from high resolution representation to a representation with half the resolution along each axis and a prolongation operator $\mathcal{P}$ which moves from low to high resolution. The restriction operation is performed by an averaging which replaces every 27 voxel neighborhood by its weighted mean. The central value is weighted by ½ and the values of all the 6-neighbors by 1/12. The prolongation is simply performed by a trilinear interpolation.

The correlation between fine and coarse grids is shown in Fig. 3 for a one-dimensional problem. The step from a fine to a coarser grid is performed by weighted averaging of every three-voxel neighborhood in three dimensions. To get to a finer grid, the value of every second voxel is determined by means of linear interpolation.

The above leads to the multi-grid algorithm proposed here:

1.   Starting with $\Phi^{(0)}$ perform i_1 Gauss-Seidel iterations for solving the equation $\mathbf{A}\Phi = \mathbf{f}$ to obtain an approximate solution $\Phi^{(i)}$.

2.   Compute the residual $\mathbf{r} = \mathbf{f} - \mathbf{A}\Phi^{(i)}$.

3.   Restrict the residual to coarse grid $\mathbf{r'} = \mathcal{R}(\mathbf{r})$.

4.   Solve for error equation $\mathbf{A'e'} = \mathbf{r'}$ on coarse grid.

5.   Prolongation of the error to high resolution $\mathbf{e} = \mathcal{P}(\mathbf{e'})$.

6.   Correct the fine grid approximate solution by the error $\Phi^{(i+1)} = \Phi^{(i)} + \mathbf{e}$.

7.   Perform i_2 fine grid iterations starting with $\Phi^{(i+1)}$.

A V-cycle is a recursion of the coarse grid correction upon itself to coarser grid. In other words, the error equation has its own error which can be solved using an embedded coarse grid correction. This is repeated for multiple resolutions. A W-cycle consists of two recursions of the coarse grid correction which means that once after step 6 it is continued with step 1 and use of $\Phi^{(i+1)}$ instead of $\Phi^{(0)}$.

To solve for the magnetic scalar potential, one W-cycle is applied with $i_1 = 5$ and $i_2 = 10$. The images are extended to a size of $(2^k + 1) \times (2l + 1) \times (2^m + 1)$. The coarsest grid used is the finest grid which contains less than 10000 voxels. On this grid size as many iterations are performed as needed for achieving convergence.

Now a possible implementation of the multi-grid algorithm will be discussed.

To solve for the boundary value problem, it is necessary to specify the value of Φ_M at boundary voxels. This is performed in two steps. A graphical representation of the procedure used for calculating Φ_M is given in Fig. 4. On the left side of Fig. 4, the original image of the object for which the susceptibility distribution is known is depicted. The imaged object was put in a larger environment and sampled down as indicated by the arrows. Then Φ_M is calculated for this sampled down image. In a subsequent step, the boundary values from the image depicted in the middle of Fig. 4 are expanded as is shown on the right side of Fig. 4. For this expanded image the Φ_M distribution is calculated next.

In the context of the procedure shown in Fig. 4, a scale factor f is determined. An object is placed in an area enlarged by a factor of f and scaled down by a factor f. The result is an image of the same size as the original but with a smaller imaged object. f should be chosen so that the object does not have any influence on the magnetic field at the borders of the new image. One may choose e.g. $f = 4$ because it can be assumed that the imaged object has no influence on the magnetic field at the borders of this environment.

Now the values of Φ_M are determined for the boundary. Assume the image has $n_x = 2^k + 1$ rows $\{0,...,2^k\}$, $n_y = 2^l + 1$ columns $\{0,...,2^l\}$ and $n_z = 2^m + 1$ slices

$\{0,...,2^m\}$. The scalar magnetic potential Φ_M if the main magnetic field $\mathbf{B}_0$ were undisturbed is determined by

$$\Phi_M(x,y,z) = \frac{f}{2\mu_0} \mathbf{B}_0 \, ((2^{k-1} - x)\,\Delta_x \, \mathbf{e}_x + (2^{l-1} - y)\,\Delta_y \, \mathbf{e}_y + (2^{m-1} - z)\,\Delta_2 \mathbf{e}_2)$$

(15)

with the unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$. Although the magnetic field is disturbed one may use this equation for computing the values of Φ_M at boundary voxels. Note that Φ_M is determined except for an additive constant which is chosen so that $\Phi_M = 0$ in the center of the image.

After computing the Φ_M distribution for the large image the values at positions corresponding to the boundaries in the original image are copied to their respective positions. The distribution is now computed in the original image.

In the next steps the $\mathbf{H}$ field is calculated from the Φ_M field (equation (2)) with the central difference scheme and the $\mathbf{B}$ field is calculated from the $\mathbf{H}$ field (equation(3)) thus obtained.

On the basis of the distorted generic MR data and the calculated distribution of the magnetic field $\mathbf{B}$, corrected generic MR data are calculated using a read out gradient having a gradient field strength $-G$ that is reversed with respect to the gradient field strength G used to acquire the distorted generic MR data.

Once the corrected generic MR data are known, the generic distortion parameters are simply the (transformation) parameters that correlate the distorted generic MR data and the corrected generic MR data.

From the above it has become apparent that the generic distortion parameters for each set of distorted generic MR data may be determined "off-line" and stored in the database 12 of apparatus 10 depicted in Fig. 1 prior to the acquisition of the distorted MR data of the object of interest. In other words, the correction of the distorted MR data of the object of interest does not necessitate that the specific magnetic field distribution resulting from the susceptibility distribution of the object of interest has to be calculated. This facilitates and accelerates the correction of the distorted MR data of the object of interest.

Determination of the corrected MR Data of the Object of Interest

Once the transformation parameters correlating the object of interest and the
corresponding generic object as well as generic distortion parameters calculated for
5   an appropriate gradient field strength G are known, the distorted MR data of the
object of interest are calculated.

To that end appropriate generic distortion parameters are determined as illustrated
above or are e.g. read out of a look up table (database 12) for a specific generic
10 object, for a specific gradient field strength G and for a specific phase encoded
direction. The generic distortion parameters are then applied to the distorted MR
data of the object of interest taking additionally into account the transformation
parameters correlating the object of interest (or its susceptibility distribution) and the
specific generic object (or its susceptibility distribution). This allows to correct in a
15 fast and easy manner distorted MR data of an object having an arbitrary shape using
previously generated information about a generic object.

Experimental Results

20 Figs. 5A and 5B show experimental results of an MR/CT registration. CT is an imag-
ing technology which is considered to be free of geometrical distortions and which
may thus serve as a reference standard for evaluating the correctness of MR data.
CT has the additional advantage that bones and soft tissue can be easily separated
by simple threshold mechanisms.

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In each of Figs. 5A and 5B two superimposed contours of a femur bone are depicted.
Fig. 5A shows the contour of a femur bone as obtained by CT and a corresponding
image generated using distorted MR data. The CT image and the MR image are then
registered with respect to each other. From Fig. 5A it becomes apparent that the
30 overlaid contours do not match very well. This is mainly due to the geometry and
intensity distortions inherent in the MR data.

Fig. 5B shows a similar view like Fig. 5A. However, instead of the distorted MR data
corrected MR data derived as discussed above have been used. From 5B it becomes
35 apparent that the MR image generated using the corrected MR data matches the
corresponding CT image much better than in Fig. 5A.

Fig. 6 shows curves of mutual information derived for the MR/CT registration in a region around a global maximum. A rotation around the z-axis is shown for the uncorrected (solid line) and corrected (dashed line) MR data. It becomes apparent that using corrected MR data the matching with respect to corresponding CT data is indeed improved.

"On-line" Processing of the distorted MR Data of the Object of Interest

In Fig. 7 a further approach according to the invention for correcting distorted MR data of an object of interest is depicted. In the approach depicted in Fig. 7, the magnetic field distribution is calculated "on-line" for the susceptibility distribution of the object of interest using the multi-grid algorithm discussed above. In principle, the determination of the corrected MR data of the object of interest is performed in the same manner as the generation of the corrected generic MR data discussed above with reference to the right branch of Fig. 2. Therefore, a more detailed description of the approach depicted in Fig. 7 is omitted.

Although compared to the approach depicted in Fig. 2 the approach depicted in Fig. 7 requires more computational time for determining the corrected MR data of the object of interest, the use of the multi-grid algorithm still considerably reduces the computational complexity compared to prior art approaches.

Use of the Invention in the Context of Computer Aided Surgery

Current three-dimensional navigation methods in e.g. bone surgery use CT or fluoroscopic imaging as a source for the three-dimensional image information which forms the basis for navigation purposes. Hitherto there have been two reasons for not using MR images. Firstly, contrary to MR images CT images are free of distortions. Secondly, bones can easily be segmented automatically in CT images. Due to lack of information on the cortical bone layer, an automatic segmentation is very difficult in MR images. The reason therefore is the fact that the cortical bone layer, i.e. the outer layer of the bone, has a low proton density and is thus difficult to distinguish in MR images.

According to the invention a navigation method is now proposed in which MR data which have been generated e.g. as depicted in Fig. 2 or Fig. 7 and which show the internal part of a bone (bone without cortical surface layer) are co-registered with fluoroscopic images. Co-registration requires very accurate information on the shape

of the structures involved. Given the method described above for distortion correc-
tion which is applied to the distorted MR data prior to co-registration, the co-
registration can advantageously be performed based on the mutual image informa-
tion obtained by MR and CT.

In Fig. 8 a navigational infrastructure 20 according to the invention is depicted. As
becomes apparent from Fig. 8, on an operating table 22 an anatomical object in the
form of a patient's femur bone 24 to be treated operatively is disposed. The surgical
intervention is performed using a surgical instrument 28 to which a marker 26 of a
tracking system is attached. The marker 26 is configured to transmit e.g. infrared
radiation. The infrared radiation emitted by the marker 26 is detected by an infrared
detector 30 disposed in a positionally fixed manner.

A localization computer 32 calculates the current spatial position of the surgical
instrument 28 from the infrared signals transmitted by the marker 26. The calculated
spatial coordinates of the surgical instrument 28 are sent from the localization
computer 32 via a databus to a central computer 34. The central computer 34
processes the spatial coordinates of the chirurgical instrument 28 to correlate these
spatial coordinates with the corrected MR data generated by the apparatus 10 as
explained above.

The apparatus 10 receives the corresponding information from the central computer
34 and displays on its display device 18 an image generated using the corrected MR
data of the object of interest, i.e. the femur bone 24, and superimposes the relative
position and the form of the chirurgical instrument 28 for navigational purposes.

Fig. 8 further shows an optional CT or fluoroscopic imaging system 36 having a C-
shape support arm 38. Disposed at one of the two ends of the C-shaped support arm
38 is an X-ray source 40 and at the opposite end an X-ray detector 42. Two-
dimensional or three-dimensional images prepared prior to or during a surgical
intervention by the imaging system 36 are fed via a databus to the central computer
34 in digitized form.

With the help of the CT or fluoroscopic imaging system 36 the navigational infra-
structure 20 of Fig. 8 generates CT or fluoroscopic images that are co-registered with
the corresponding MR images derived from the MR data of the femur bone 24. This
allows to display on the display device 18 co-registered MR and CT or fluoroscopic

images simultaneously with the current position of the surgical instrument 28 with respect to the co-registered images.